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RELIABILITY BASED FATIGUE DESIGN AND MAINTENANCE PROCEDURES

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ABSTRACT

Demand for light weight aircraft structures results in the use of as small a safety margin as is practical. As a consequence of the small safety margin and other uncertainties, cracks or partial damages are likely to occur before the economical life of the aircraft is expended. Fatigue is one of the principal causes for the cracks. Fatigue loading and fatigue crack growth also contain uncertainties.

The susceptibility of the aircraft structure to crack or partial damage during the useful life of the structure imposes the requirement that the structure should be capable of supporting the service loads with these cracks. Furthermore, it must be possible to detect these cracks before they extend to critical sizes and cause catastrophic failure of the structure. Therefore, any fail safe design that can achieve this objective needs a knowledge of the probability of the presence of a crack of a certain length at a given location after certain number of flight hours. A stochastic model has been developed to describe such a probability for fatigue process by assuming a varying hazard rate. This stochastic model can be used to obtain the desired probability of a crack of certain length at a given location after certain number of cycles or time.

Quantitative estimation of the developed model has also been discussed. Application of the model to develop a procedure for reliability-based cost-effective fail-safe structural design has been discussed. This design procedure includes the reliability improvement due to inspection and repair. Methods of obtaining optimum inspection and maintenance schemes have also been discussed.

Alternate methods of fatigue reliability improvement by cold working processes have been discussed. The associated stress corrosion problem has been studied. Application of statistical decision theory to select suitable test options and safety factors subject to a reliability constraint have also been investigated.

Most of the investigations under this project have either been published in journals and conference proceedings or pending publication.

INTRODUCTION

It is now generally accepted that all structural materials are not "flaw-free". Sometimes, a maximum acceptable flaw-size can be specified as a part of the structural specifications. Thus, an initial flaw size (a_o) and the associated probability distribution characterize the structure. Due to fatigue loading, these initial "micro-sized" flaws grow to detectable sizes. Time or number of cycles required for this growth to detectable size of crack length is often called the "crack initiation time." In many cases, this growth time amounts to a significant function of the total fatigue life of the structure. Due to further fatigue loading, crack sizes increase until they attain critical sizes. These critical sizes depend upon the critical stress intensity factors and the external loads. Thus, the probability distribution for crack sizes is changing continuously with time or number of cycles at all locations of the structure. Therefore, the probabilistic description of fatigue process can be expressed as the probability that for time $t \leq T$, the crack size $a \leq A$. This is a shochastic process.

In most of the reported works [1-8], the varying crack lengths "a" associated with the fatigue process are ignored. In these works, stochastic process is not considered. The entire fatigue process is described by a single random variable "t" which is the time for fatigue failure. The quantitative description consists of the probability that for time $t \leq T$ fatigue failure will take place. Because of the simplicity of the

model, probability distributions such as the Weibull Distribution have been used to describe the time for fatigue failure.

The use of such a description that relies on a single random variable is very limited because such a model neglects many important aspects of fatigue process. For example, one question that needs an answer is as follows. What is the length of the crack that corresponds to the defined failure time? Is this length the initiation length or critical crack length or some arbitrarily chosen length? Initiation length can vary depending on the available non-destructive inspection capability. Furthermore, such a model does not provide any information for optimizing repair threshold crack length, crack arresting devices, N.D.I. capabilities and different loading process. Another arguement used by the users of a single random variable is to assume that the effect of varying crack length is negligable and a stochastic process is not needed. In order to verify if such a statement could be true fatigue data from specific fleet of aircraftare analyzed as a first step of the investigation. As explained in later sections, these investigations demonstrated that a stochastic model is necessary to describe fatigue of structure .
Further investigations during the project period are described as follows:

a) Development of a simple stochastic model for fatigue by using the concept of a varying hazard rate and a birth process[9-10],

b) Quantitative estimation of the parameters of the stochastic model by using fracture mechanics considerations[9].

c) Quantitative estimation of the parameters of the stochastic model from available data[11,12].

d) Application of the stochastic model to develop a reliability-based, cost effect fail-safe design procedure[9].

e) Development of procedures for devising optimum inspection and maintenance schemes [11, 12].

f) Application of statistical decision theory to select appropriate test options and safety factors subject to reliability restraints[13].

g) Investigation of alternate methods of improving fatigue life and fatigue reliability by using interference fit techniques and the associated stress corrosion considerations[14,15].

h) Application of the principles of analysis of variance to study the significance of present methods of grouping fatigue failure data[16].

As a by-product of the above investigations, an improved mathematical technique has been developed. This technique and its application can be described as follows:

i) An improved numerical technique of multiple integration with respect to one independent variable[17].

j) Application of new technique of integration to develop a procedure for the study of some random vibration problems[18].

Analysis of Fatigue Failure Data

In order to investigate if fatigue provess can be described by a single random variable "t" that denotes time for fatigue, fatigue data from two specific fleets were analyzed. A typical inspection record contained the following information.

1. Identification number of the airplane
2. Number of flight hours completed before the inspection
3. Inspection date
4. Number of reinspection(s)
5. The command
6. The base
7. Facility of inspection
8. Crack location by numbers of the critical regions as has been previously identified
9. Number of such cracks in a given region
10. Direction of crack growth
11. Crack length
12. Information as to whether the crack has been repaired

A two parameter weibull model was hypothesized for the fatigue failure time "t".

$$F(t) = 1 - \exp\left[-(t/\beta)^{\alpha}\right] \qquad (1)$$

In the equation, α and β are shape and scale parameters respectively. These parameters were estimated from the data by using the method of maximum likelihood[19].

The chi-square and Kolmogrov[20] tests were used to verify the goodness of fit of the estimated parameters. The following conclusions were reached.

(1) For a given critical location or a selected group of critical locations, no acceptable Weibull distribution was obtained unless the data were censored in some way. In general, censoring of both high-level outliers and low level outliers were needed. Low-level outliers refer to those fatigue failure times that lie below a selected failure time for purposes of censoring. Similarly, high-level outlier refers to those failure times that lie above a time corresponding to high-censoring level. Use of a low level outlier censor was not conservative. Any model derived by the use of low level censoring can result in serious errors in decisions concerning design and maintenance. Similarly, models derived by the use of high level censoring can result in increased weight and cost.

(2) A three parameter weibull distribution or a log-normal distribution did not improve the results.

(3) However, when the observed failure times at a given location were reduced by regression techniques to correspond to the time for initiation of crack of a given length acceptability of the two-parameter weibull model improved in many cases. Probability distribution was different for different crack lengths. Necessity for a stochastic model was evident.

Development of A Stochastic Model for Fatigue

It is assumed that a single crack is present in a fatigue critical region. Multiple cracks can be treated by order statistics or other procedures. Then, the variation of crack length with time is qualitatively of the type shown in Figure 1. This consists of a continuous variation of crack length with continuous variation of time or number of cycles. The corresponding model for the stochastic process for fatigue crack sizes involves the consideration of continuous state space of crack lengths and continuous time. It is difficult to develop such a model. The development of the model is simplified by considering the state space of crack length to be discrete as shown in Figure 1. Accuracy can be increased by decreasing the magnitude of '$\Delta\ell$' of discrete crack length increments. This process of considering the state space of crack length can also accomodate consideration of crack initiation i.e. probability of a crack of length a_i initiating at time t less than or equal to t_i as shown in Figure 2. Even though the crack lengths are assumed to increase in discrete steps the mean crack growth rate can vary continuously as a function of time. Because the resulting process is nonstationary, the probability that a crack of length $k(\Delta\ell)$ i.e. , k times $\Delta\ell$, is present at a time $t \leq t_k$ depends on the initial value of time t_o. This is denoted by $P(k, t_o, t_k)$.

By considering the different ways in which the event of the development of a crack of length $k(\Delta\ell)$ can occur in time interval t_o to $t + \Delta t$ the following equation can be written

$$P(k, t_0, t+\Delta t) = \sum_{i=0}^{\infty} P_i(k, t_0, t)\, P(k-i, t_0, t+\Delta t) \qquad (2)$$

by assuming orderliness of crack growth i.e.,

$$\lim_{\Delta t \to 0} \frac{S_2(t, t+\Delta t)}{\Delta t} = 0 \qquad (3)$$

and

$$\lim_{\Delta t \to 0} \frac{S_1(t, t+\Delta t)}{\Delta t} = 0 \qquad (4)$$

where

$$S_1(t) = \sum_{k=1}^{\infty} P(k, t_0, t) = 1 - P(0, t_0, t) \qquad (5)$$

and

$$S_2(t) = \sum_{k=2}^{\infty} P(k, t_0, t) = 1 - P(0, t_0, t) - P(1, t_0, t) \qquad (6)$$

It can be shown that the following differential equations are for $k > 1$

$$\frac{\partial}{\partial t}\left[P(k, t_0, t)\right] = E[\dot{a}(t)]\, P(k-1, t_0, t) - E[\dot{a}(t)]\, P(k, t_0, t) \qquad (7)$$

In this equation, $E[\dot{a}(t)]$ is the mean crack growth rate at t. For $k = 1$, the equation (7) takes the following special form.

$$\frac{\partial}{\partial t}\left[P(1, t_0, t)\right] = f_e(t) - E[\dot{a}(t)]\, P(1, t_0, t) \qquad (8)$$

where $f_e(t)$ is the probability density for crack initiation. These equations can be solved by methods similar to those discussed in Reference (11). However, P(k, t) can be obtained only if $E[\dot{a}(t)]$, $f_e(t)$, and the probability distribution for initial flaw sizes are known. This procedure will be discussed in a separate note. The method of obtaining $E[\dot{a}(t)]$ is discussed in the next section.

Mean Crack Growth Rate

Knowledge of the mean crack growth rate is essential to estimate the crack length at a given time. According to Forman[21] the rate of crack growth is given by

$$\frac{da}{dN} = \frac{C_1 (\Delta K)^n}{(1-r) K_{1c} + \Delta K} \qquad (9)$$

where C_1 and n are material constants, ΔK is the range of stress intensity factors, K_{1c} is the critical-stress intensity factor, r is the ratio of minimum stress intensity factor to the maximum stress intensity factor 'a' is the half crack length and 'N' is the number of cycles. For a stiffened panel the range ΔK is given by

$$\Delta K = \Delta L \left(\pi a \cdot f\left(\frac{a}{w}\right)\right)^{\frac{1}{2}} C_R(a, b) \qquad (10)$$

where ΔL is the range of applied loads at a given time, $f(\frac{a}{w})$ is the finite width correction factor, C_R (a, b) is the tip stress reduction factor, and b is half the stringer spacing. For a fixed value of 'a', $\frac{da}{dN}$ is a function of the random load parameters ΔL and r. Thus at a given crack length say a - a_1, the growth rate is a random variable. The expected value of teh growth rate is given by

$$E\left[\frac{da}{dN}\right]_{a=a_1} = \int_{R_{\Delta L, r}} \int \left(\frac{da}{dN}\right)_{a=a_1} f(r, \Delta L)\, dr\, d(\Delta L) \qquad (11)$$

where f(r, ΔL) are the density functions of the random variables r and ΔL respectively. $R_{\Delta L, r}$ is the range space for ΔL and r respectively. Equation (11), thus gives the mean crack growth rate at any value of crack length under the random loading. This quantity expressed in terms of the discrete length units ΔL is required in the equation for P(k, t) of previous section

The mean crack growth rate as given by Equation (11) is a complicated integral to be solved and does not have a closed form solution. Hence, numerical methods have been used to solve the equation. However, for a special cases where r and ΔL are stationary Gaussian processes, Taylor's series expansion has been used to obtain approximation. Then $E[\dot{a}]$ at any value of a_i is given by the following equation.

$$E\left[\frac{da}{dN}\right]_{a=a_i} = A\mu_{\Delta L}\left\{\mu_{\Delta L}^2 + 3\sigma_{\Delta L}^2\right\}\Big/\left[B(1-\mu_n) + C\mu_{\Delta L}\right] - 3A\cdot C\cdot\mu_{\Delta L}^2\sigma_{\Delta L}^2\Big/\left[B(1-\mu_n) + C\mu_{\Delta L}\right]^2 + A\mu_{\Delta L}^3\left\{C^2\sigma_{\Delta L}^2 + B^2\sigma_n^2\right\}\Big/\left[B(1-\mu_n) + C\mu_{\Delta L}\right]^3 \quad (12)$$

Alternate Method of Estimation of Parameters

An alternate method of estimation of parameters is to use the fatigue failure data from the same fleet, similar fleet or from tests. Such a method requires the following steps.

(1) The first required step is the solution of equation (7) and (8) to obtain P(k, t). This could be left in the form of quadratures.

(2) The next step needed is the normalization to a realistic maximum crack length N (ΔL).

(3) Next step is to estimate the parameters. This has been done by using maximum likelyhood method and by using loss function concept in decision statistics.

This work has been carried out as a part of the project investigation. Preliminary results are published in references 11 and 12. These papers

include the consideration of data from a specific fleet supplied by NASA. A detailed analysis including the model verification will be published[12].

Applications of the Developed Model

One of the applications of the developed model is to develop a reliability-based, cost-effective design procedure. This method has been developed and reported by the investigators in reference 18. Some of the significant items and example problems are discussed here.

Problem Setting

The problem setting can be best explained by considering an example. In this report, the deisgn of a built-up structure such as a sheet-stiffener combination is considered. Figure 3 illustrates the stiffened panel. The panel is of width w and thickness t. The panel is assumed to be made of a specific material and the particular structure is assumed to be a sub-assembly of an aircraft structure. It is also assumed that large number of aircraft will be produced as a result of this design. Even though the discussed methodology considers a specific material, an optimum choice among several candidate materials can be made by following a similar procedure and statistical decision theory. External loading F consists of a sustained loading F_1, and a random fatigue loading F_2. It is assumed that the random fatigue loading has been quantified probabilistically. Thus, the total loading F is specified probabilistically.

For a particular choice of the thickness t, the stringer spacing 2b, and the choice of the material, the initial ultimate load carrying capacity F_u is known. If the initial microsized flaws or cracks are

specified by a probabilistic distribution, the initial load carrying capacity F_u is characterized by an appropriate probabilistic distribution which depends on the initial flaw size distribution, the material and the dependence of the load carrying capacity of the structure on the flaw size and other dimensions.

On the other hand, if it is assumed that the effect of initial flaw size distribution can be described by a crack initiation probability distribution, the load carrying capacity F_u can be expressed as a deterministic quantity if the material properties are also assumed to be deterministic. The corresponding initial ultimate stress is defined to be σ_u. Similarly, for a given thickness, stress corresponding to external loading is denoted by σ_L. If σ_u and σ_L are deterministic, the initial safety margin i.e., before fatigue effects are present, is given by the ratio of σ_u to σ_L. As explained earlier, both σ_u and σ_L have uncertainties and need probabilistic representation. Then the initial reliability can be considered as a safety measure. This can be represented by the probability that σ_u / σ_L is greater than 1. Due to the presence of fatigue loading, cracks grow in size. Crack growth rates and the crack sizes depend on the material properties, stress and the number of cycles. The presence of a crack of size a_i reduces the ultimate strength from σ_u to σ_{ui}. Then the reliability which is defined by the probability that the ratio σ_{ui} to σ_L is greater than 1 is also reduced. Consequently, the probability of failure which is the probability of the ratio $\sigma_{ui} \geq \sigma_L$. The probability of failure increases as teh crack lengths increase. The probability of failure can be reduced by increasing the initial margin

of safety or reliability. This of course, increases the weight of the structure. Another way of decreasing the probability of failure is to inspect the structure at selected times so that the cracks can be detected and repaired before they reach their critical sizes. In this process, allowable initial margin of safety can be small because cracks are not allowed to grow to their critical sizes. This process however, increases the cost due to inspection. Increasing weight also increases the initial cost and the cost of operation. Therefore, the required design procedure consists of selecting the design variables such as the thickness, stiffener spacing, and inspection frequency during the projected service life so as to minimize the total expected cost or weight. The cost and weight can be considered as interchangeable functions that can be optimized. Many a time it is easy to express the objective function to be optimized as an equivalent weight function. This entire procedure, however, is subjected to the restraint that the margin of safety or reliability does not fall below an acceptable limit during the projected life of the structure.

Therefore, reliability-based fail-safe fatigue design procedure consists of selecting specified design variables including inspection frequency, subject to constraints, so as to minimize the expected cost or weight function while the probability of failure is kept below specified limits during the projected life of the structure. In order to make the design procedure acceptable to a designer who is not familiar with the statistical methods, the reliability or probability of failure can be related to a 'variable' safety factor or safety margin.

Methodology

The following are the steps that need to be followed in the methodology for the reliability-based fail-safe fatigue design procedure discussed in this paper.

- The first step consists of specifying the design variables and constraints. This step identifies the design variables that can be selected by the designer to minimize the objective-function (weight or cost).
- The second step is to specify the probabilistic distribution of the external loading. This can be a stochastic process.
- The third step is to formulate the objective function. This can be a weight or cost function and is related to the probability of failure, the projected life of the structure, the specified and selectable design variables, and external loading.
- The fourth step is to select trial design variables and obtain the initial margin of safety or reliability.
- The next step is to obtain the variation of crack size and crack growth probabilities with time. A stochastic model for crack growth developed by the authors is used in this report to obtain the probabilistic description of crack sizes. This probability depends on the material, load description and the number of cycles.
- From this knowledge of the probability distribution of crack sizes, reduction in strength and probability of failure is estimated. The inspection and repair frequency during the projected design life is included in this estimate of the probability of failure.

. The seventh step is to substitute all the information in to the cost or weight that was formulated in the third step. This yields the cost or weight due to the particular selection of the trial design variables.

. Steps two to seven are repeated with different trial variables to minimize the objective function by search method.

. The final design variables are selected subject to restraints such as reliability bounds, minimum spacing, etc.

These are the general steps that are necessary in the design procedure developed in this report. This needs the description of a stochastic model for fatigue crack growth and crack sizes, methods of estimation of the probability of failure, methods of including the effects of inspection and repair frequency during the projected design life in the probability of failure, and an objective function in terms of cost or weight. The stochastic model and the estimation of the parameters of the model are already discussed in previous sections. The estimation of the probability of failure, reliability improvement due to inspection and repair, formulation of the objective function and its minimization are discussed in the following sections.

Probability of Failure

In this section, method of estimating probability of failure is discussed. The improvement in reliability due to inspection, repair and consequent renewal and the estimation of this reliability improvement are not discussed in this section. These are discussed in the next section.

The first step in estimation of probability of failure is to identify the possible failure modes. In addition to the fatigue failure mode, other failure modes such as the sudden over stress or buckling are possible. If the event of fatigue failure is denoted by E_f, the event of sudden over stress by E_s and the event of buckling failure by E_b. The probability of failure P_f is given by the union of the all the possible events of failure.

$$P_f = P[E_f \cup E_s \cup E_b] \tag{13}$$

Probability of occurrence of each of these events depends on the strength of the structure to resist that particular type of failure and the probability of occurrence or the load that can result in that particular type of failure. Because the discussions of the paper are primarily restricted to fatigue failure, it will be assumed that only fatigue failure are possible. This means that only failure mode possible is due to the growth of fatigue cracks and consequent reduction in strength.

Before discussing the probability of failure under conditions of uncertainty, a deterministic design procedure is briefly reviewed here. This review is useful in identifying the different probabilistic fatigue failure modes. Consider the stiffened panel shown in Figure 3. Let it be assumed that a central crack is likely to develop in this structure due to fatigue. For given w and assumed length between stiffeners 2b, the variation of the residual strength σ_u with half the length of the central crack[29-31] is shown in Figure 4. The value of the maximum external load L is precisely known in deterministic design. Then, for a particular choice of the initial safety margin S, the thickness t and

the corresponding stress σ_L, critical crack length a_c can be obtained. These are shown in Figure.4. As the fatigue cracks initiate and grow, failure is not possible until the crack attains a length of a_c. The length of a_c can also be obtained analytically from the following formula in the case of a stiffened panel.

$$K_c = \frac{L}{t}\sqrt{\pi a_c}\, f\left(\frac{a_c}{w}\right)\, C_R(a_c/b) \qquad (14)$$

In this equation $f(a_c/w)$ is the width correction factor[22], C_R is the tip stress reduction factor[23], K_c is the fracture toughness of the material.

Because the maximum load L is known precisely in a deterministic case, the stresses due to external load never exceed the residual strength for crack lengths $a < a_c$. Alternately, it can be stated that probability of failure is zero for crack lengths $a < a_c$ and the probability of failure is one for $a \geq a_c$.

In reality, the external load is not precisely known. The load is usually characterized by a random variable. This is the case in which reliability based design procedures are needed. In this paper, external loading is assumed to be characterized by a stationary stochastic process. Even in this case, a value of a_c can be selected in the Figure 4. This curve is assumed to be known deterministically. This means that for a given width of the panel w and a choice of stiffener spacing 2b, a value of critical crack length a_c is chosen. This value of a_c corresponds to a definite value of σ_L on the curve in Figure 4. But, the external loading is not known precisely as in the deterministic case. Therefore, the value of a_c and σ_L cannot be related to initial safety margin and choice

of thickness t. However, the probabilistic description of the external loading L is known. As will be shown later, the choice a_c, σ_L, and thickness t can be related to reliability or probability of failure. From a knowledge of the specified bounds on reliability, a_c and t can be chosen.

Alternately, the following procedure can be used instead of starting with a choice, a_c. A value of $\bar{\sigma}$ is selected such that

$$\bar{\sigma} = \mu\left(\frac{L}{t}\right) + \bar{\alpha}\,\Sigma\left(\frac{L}{t}\right) \qquad (15)$$

where $\mu\left(\frac{L}{t}\right)$ is the mean value of external load divided by the choice of thickness t and $\Sigma\left(\frac{L}{t}\right)$ is the corresponding variance. The quantity $\bar{\alpha}$ is constant which is similar to safety margin in a deterministic design. However, $\bar{\alpha}$ is not arbitrary. The quantities $\bar{\alpha}$, t and a_c are related to reliability. They can be selected on the basis of the prescribed reliability bounds. As can be seen in the figure, a selected value of corresponds to a value of which corresponds to a value of a_c.

Unlike the case of deterministic loading, failure may take place even for values of crack sizes smaller than a_c. Such a failure is possible because the externally induced stress (L/t) has a probability distribution and does not represent the absolute maximum possible stress. For values of $a < a_c$, fatigue failure is possible if the externally induced stress exceeds the residual strength at any time during the service life of the aircraft. This failure is defined as static fatigue failure P_{sf}.

In order to simplify the procedure for estimating the reliability, the concept of critical crack size fatigue failure has been introduced.

At any time during the service of the aricraft, the crack size at a given location can be either greater than or equal to a specified length a_c or less than a_c. Then, the following two mutually exclusive events can be defined.

(a) failure occurs when the crack length at the location is less than a_c

(b) failure occurs when the crack length at the location is greater than or equal to a_c.

The event (a) has been denoted by p_{sf}, the event (b) will be denoted by p_{fc}. Then p_f can be written as follows.

$$P_f = P_{fc} + P_{sf} \tag{16}$$

In this equation

$$P_{fc} = P(K \geq K_c, t \mid a \geq a_c, t) \tag{17}$$

In many cases, for practical reasons, a particular value of critical crack length can be defined. The structure is considered to have failed if the crack length at a given location exceeds this value. For example, this absolute critical crack length can be the critical length corresponding to the sustained loading F_1. Then

$$P_{fc} = P(a \geq a_c, t) = P(k\Delta l \geq k_c \Delta l, t) \tag{18}$$

In this case P_{fc} can be called critical size fatigue failure.

Reliability Improvement Due to Inspection and Repair

If no inspections are done during the projected design life, the probability of critical crack size fatigue is given by

$$P_{fc} = P(a \geq a_c, t) = P[k \geq k_c, t] = \sum_{k=k_c}^{N_{max}} P(k\Delta\ell, T_D) \qquad (19)$$

In this equation N_{max} is a number that has been defined during the normalization of the probability distributions. As pointed out earlier probability distributions are normalized to a realistic maximum crack length. The reason for normalization is that cracks in reality do not grow to infinite lengths. Then,

$$N_{max} = \frac{\text{maximum crack length}}{\Delta\ell}$$

The quantity T_D, in equation (19), is the projected design life of the sturcture. The probability of critical crack size fatigue fialure P_{fc} can be improved due to inspections. This change in probability of failure and hence in reliability can be obtained in the following way.

The projected design life is still assumed to be T_D number of hours or cycles. It isassumed that one inspection is done at T_o number of hours or cycles. It is further

assumed that this inspection is conducted at $T_o = 0.5\ T_D$. At the time of inspection, if cracks of length $k(\Delta L) \geq k_r(\Delta L)$ are observed, the cracks are repaired. The quantity $k_r(\Delta l)$ is the repair threshold crack length. It is further assumed that structure is as good as new after repair. This means any further crack initiation and growth are to be calculated as though the structure is put into service at $t = T_o$ and not at $t = o$. It is also to be noted that only structures with $k_r < k < k_c$ are repaired because the structures with $k(\Delta l) \geq k_c(\Delta l)$ have failed due to critical size fatigue failure. It is implicit that the cracks of $k(\Delta l) < k_r(\Delta l)$ are not repaired.

There is still another quantity to be considered. This is the probability of detecting a crack by nondestructive inspection techniques if a crack exists. In the first step of the derivation, it will be assumed that the repair threshold crack length $k_r(\Delta L)$ is chosen that the detection probability is one. Then, the probability of critical size fatigue failure in the two intervals can be obtained as follows.

The probability of failure P(1) in the first interval corresponding to $0 < t \leq T_o$ is given by

$$P(1) = \sum_{k=k_c}^{N_{max}} P[k(\Delta l), T_o] \qquad (20)$$

By referring to Figure 4, the probability of survival in $0 < t < T_o$ is 1 - P(1) because there is the probability P(1) that structures fail in $0 < t \leq T_o$. For $t \leq T_o$,

$$P[k < k_R, T_o] + P[k_R \leq k \leq k_c, T_o] + P[k \geq k_c, T_o] = 1 \qquad (21)$$

and the probability of repair P_R is given by

$$P_R = P[k_R \leq k < k_c, T_o] \qquad (22)$$

Then the total probability of critical crack size fatigue failure in $0 < t \leq 2T_o = T_D$ can be written as follows:

$$P_{fc} = P(1) + P_R P(1) + F_1 [P(2) - P(1)] / [1 - P(1)] \quad (23)$$

where $$F_1 = P[k < k_r, T_o] \quad (24)$$

and $$P(2) = P[k \geq k_c, 2T_o]$$

Equation (23), for the probability of failure under one inspection is obtained by considering the three mutually exclusive and exhaustive events F_1, P_R and P(1) [see Equation (21)]. The quantity in the parenthesis of the last term of Equation (23) is the conditional probability that the structures will fail in $T_o < t \leq 2T_o$ given that they survived $0 < t \leq T_o$. This expression for P_{fc} satisfies all the limiting conditions. For example, when $P_R = 0$, P_{fc} reduces to P(2), as expected.

When $P_R = 1$ and hence P(1) = 0, P_{fc} becomes zero. Similarly, the probability of failure under any number of inspections can be obtained.

If the crack detection probability due to nondestructive inspection techniques is considered, the probability of repair P_R changes. The repair is now possible only if a crack of size $k_r(\Delta 1) < k(\Delta 1) \leq k_c(\Delta 1)$ exists and is detected by the NDI capability, with a probability D(k). Here, D(k) is the probability of detecting a crack of size k(1). D_k depends upon the NDI accuracy. A representative function for D_k is assumed as follows.

$$\Delta 1 \; D_k = 0 \qquad a < a_1$$

$$= \frac{a - a_1}{a_2 - a_1} \qquad a_1 \leq a \leq a_2$$

$$= 1.0 \qquad a \geq a_2$$

In the illustrative problem, it is assumed that $a_1 = 0.02"$ and $a_2 = 0.3"$.

Then, the unconditional probability of detecting and repairing cracks of size $k_r(\Delta L) < k(\Delta L) \leq k_c(\Delta L)$ at T_o is given by

$$\bar{P}_R = \sum_{k=k_m}^{k_c-1} P[k, T_o]\ D(k) \qquad (25)$$

Then, of the repairable aircraft given by $P[k_r \leq k \leq k_c,\ T_o]$ only $\bar{P}_R$ are repaired and the others are not repaired. Now, equations similar to (23) can be written with detection probability for cracks included.

Total Weight Function

Every optimization problem involves the so-called objective function which is a function of the design variables appropriate to the problem at hand[24-28]. The optimum values of the design variables are obtained by finding thestationary locations of the objective function subject to the design constraints[24-28].

For aircraft structures "weight" is the most crucial consideration in design. In the present context, the weight of the stiffened panel is considered to be minimized. The design variables are the thickness of the sheet and the width of the stringer spacing. The total "weight function" comprises of the deterministic weight of the panel and the expected gain of weight is given by the product of the probability of failure under a given number of inspections and the deterministic weight of the panel. The deterministic weight of the panel consists of the weight of the sheet and the stringers. Expressed mathematically, the total weight function is given by

$$W(b,t) = (1 + P_f)[wthy + N_{st} W_{st}] \qquad (26)$$

where w = total width of the sheet

t = thickness of the sheet

h = breadth of the sheet

γ = density of the sheet material

N_{st} = number of stringers

$2b$ = stringer spacing

W_{st} = weight of one stringer

Equation (26) is the proper objective function for the minimization of the weight. The effect of increasing the thickness is to reduce to expected loss of weight because of the reduction in the probability of failure. On the other hand, the deterministic weight is increased by increasing the thickness. Thus, a balance has to be found between the two. Stringer spacing has the opposite effect on the different weights.

The minimization is carried out by the search method. The total weight function is calculated for a set of thicknesses and stringer spacings. It is then plotted versus thickness with stringer spacing a the parameter. Then, the lowest weight is selected. The thickness and the stringer spacing corresponding to the minimum weight are the optimum values if the reliability constraint is satisfied at these values. The weight can be expressed in terms of equivalent costs.

Total Cost Function:

If the problem at hand is the determination of the optimum number of the periodic inspections, then the total weight function may not be the proper objective function. Then the total cost function concept has to be introduced. The total cost function comprises of the expected cost of failure and the deterministic cost of the periodic inspections. The expected cost of failure is given by the product of the probability of failure under the given number of inspections and the deterministic

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cost of structure. The deterministic cost of inspections is proportional to the number of inspections. The mathematical expression for the total cost function is given as follows:

$$C_T(j) = P_f C_s + JC_I \qquad (27)$$

where P_f is the probability of failure under j inspections

C_s is the cost of new structure

C_I is the cost of one inspection

J is the number of inspections

Equation (27) gives the proper objective function because as the number of inspections increases, the expected cost of failure decreases while the cost of inspections increases. The minimum value of the total cost function is found by the search method. The minimization is subject to the reliability constraint.

Illustrative Example

In order to illustrate the developed method, two examples have been considered. The first problem is that of a minimum weight design of 7075-T6 alloy. The problem has been deliberately kept simple for purposes of illustration. A more detailed problem is discussed in the Appendix II.

The design life is supposed to be 15,000 cycles with two periodic inspections made during the design life. The reliability is to be 99.5%. The design variables to be selected are the thickness t and the spacing of the stringers 2b. The following data is assumed to be known.

w =	0.61m	C_1 =	5×10^{-13}
α =	8.9	h =	0.3048m
β =	5000 cycles	$\sigma^2_{\Delta L}$ =	307×10^4 (N^2/m^2
$\mu_{\Delta L}$ =	175200 N/m	σ_r^2 =	0.01
μ_r =	0.5	W_{st} =	0.05 Kg
γ =	46.39 Kg/M^3	n =	3
K_{IC} =	$7475/.23 \times 10^3$ $N/m^{3/2}$		

α : shape parameter of Weibull distribution for distribution for crack initiation

β : scale parameter of Weibull distribution. The probabilistic description of the loading is given in terms of the mean and variance (σ^2) of the load range ΔL and load ratio r.

As outlined in the preceding sections, the solution procedure is carried out. As a first step, the residual strength-critical crack length diagrams are obtained for a choice of number of stringers, e.g. 3, 5, 7, 9, 11, etc. (Fig. 6). As the number increases the stringer spacing decreases. As one might expect, the rate of growth decreases with the number of stringers. The tip stress reduction factor $C_R(a/b)h$ which is required in the expression for the residual strength is obtained from references (29-30) as shown in Figure 7.

The variation of the static reliability with residual strength and thickness is shown in Figure 8. For a given loading, in order to maintain the same static reliability, the thickness has to increase as the design residual strength decreases and vice versa.

In Figure 9, the relation between the probability of static failure, fatigue failure, and total fialure is delineated.

The total weight functions are calculated in the manner explained previously for fixed $R_S = 0.9996$ and $N_{stringer} = 3, 5, 7, 9$, Figure 10 depicts the minimization curves. From these curves, the minimum W for each curve can be obtained, and then compared with other minima of other curves. The overall minum in Figure 10 occurs for a thickness of 0.106 inches, $N_{stringer} = 7$.

Figure 11 represents the minimzation curves for $R_s = 0.9997$. As expected, the minimum values are now changed, and occur at different thicknesses. The minimum now occurs for $N_{stringer} = 7$ and thickness $t = 0.1044$ inches. From Figure 12, for $R_s = 0.9998$, the overall minimum decreases to 3.554 and at $N_{stringer} = 7$ and $t = 0.1052$.

Then the static reliability R_s is increased further to $R_s = -.9999$ the overall minimum is higher than before, i.e. $W_{min} = 3.630$ and occur for $N_{stringer} = 7$ and thickness $t - 0.1052$, Figure 13.

Thus comparing all the minima over the various variables, the minimum most is $W_{min} = 3.554$ for $R_s = 0.9997$, $t = 0.1044$ inches and $N_{stringer} = 7$. R_s, the static reliability is equal to $(1-P_{fs})$, where P_{fs} is the static fatigue failure probability. The relation between P_{fs} the critical crack fatigue fialure and P_f the total probability of fialure is as follows

$$P_f = P_j + (1-P_j) P_{fs}.$$

This is plotted in Fig. 9.

$R_s = 0.9996$ does not lead to minimum because (1) the corresponding thickness is higher than for $R_s = 0.9997$.

(2) the effect of lower R_s (0.9996) implies that the structure is being designed for a lower residual strength (design stress) from figure 8. This figure is the plot of the following equation not quoted in the report.

$$R_s = \frac{\left[t\sigma_R \left(\frac{1}{\mu_L} + \frac{\sigma_L^2}{\mu_L^3}\right) - 1\right]^2}{t^2\sigma_R^2 \frac{\sigma_L^2}{\mu_L^4} + \left[t\sigma_R \left(\frac{1}{\mu_L} + \frac{\sigma_L^2}{\mu_L^3}\right) - 1\right]^2}$$

where σ_R is residual strength

μ_L is the mean of the load

σ_L^2 is the variance of the load

t is the thickness.

The lower design residual stress results in a higher P_j by virtue of longer critical crack length (see Fig. 6). The interaction of all these results in a higher total weight function for $R_s = 0.9996$. It is to be noted that W_{total} is not the deterministic weight of the panel. This corresponds to an overall reliability of 0.99765 and a design residual strength = 15,500 psi. The reliability constraint is satisfied since 0.99765 is greater than the reliability bound $R_b = 0.995$.

Check on the Initial Factor of Safety:

The mean and standard deviation of the maximum load L_{max}, are obtained. Then, considering different numbers of standard deviations above the mean-maximum load L, the initial factors of safety are obtained. For example, for one standard deviation above L, the initial factor of safety of the optimum design, based on yield

strength is found to be 3.067. When two and three standard deviations are employed, the corresponding factors of safety are 2.60 and 2.32 respectively. This provides a comparison of the optimum design values and the values from equivalent deterministic designs.

Cost Optimization:

To demonstrate cost optimization, the designed stiffened panel is considered. The only variable now, is the number of periodic inspections or the inspection interval. Since the panel is of a given configuration, its weight is fixed. Hence the total cost function C_T Equation (34) is the proper objective function to be considered in the present context.

As a first step, the probability of fatigue failure under j inspections, $j = 0, 1, 2, 3, 4, \ldots$etc. is calculated. These values are graphically depicted in Figure 13. Corresponding to each of these numbers of inspections the total cost function C_T is calculated from Equation (34), Figure 15. This is repeated for various values of the ratio of the cost of one inspection C_I to the cost of the structure C_S. When $C_I/C_S = 0.1$, the minimum occurs for one inspection. Decreasing C_I/C_S to 0.01, 0.005, 0.001 renders the minimum to occur at two inspections, three inspections and four inspections respectively as delineated in Figure 14.

Alternate Methods of Improving Fatigue Life Fatigue Reliability

The models for fatigue discussed in the preceding sections do not apply to cases for which residual stresses are present near fastener holes due to a cold working process such as stress coining. The purpose of stress coining is to improve the fatigue life of the structure. A simple method of stress coining in aluminum alloy is to expand the

fastener hole of the structural member by drawing an oversized mardel hydraulically through the fastener hole. Many similar processes are available for cold working fastener holes.

Such cold working processes result in a radial flow of the material. This results in residual stresses. Residual compressive stresses surrounding the hole provide protection against the fatigue damage by opposing the applied tensile stresses, However, as shown in the investigation, there is a zone of sustained residual tensile stresses located at a short distance from the hole. The maximum tensile stress usually occurs at the elastic-plastic boundary. Although the tensile stresses are not critical in the point of view of fatigue life of the structure, they can cause stress corrosion under certain conditions.

Therefore, the reliability of a stress coined structure needs the consideration of both the fatigue improvement and stress corrosion sustibility. The first step in such a study is to assess the residual stresses and stress coining susceptibility in such structures The investigations carried out in the project have been published in references 14 and 15.

By-Products From the Project

As a by-product of the investigations, the following have been developed. An improved numerical technique was needed in quantitative estimation of the parameters of the stochastic model. This has been discussed in Appendix III. An application of the technique has been done to random vibration problems. The purpose of the application was to verify the accuracy of the technique.

Another by-product is the application of the principles of analysis of variance to study the significance of the present methods of grouping fatigue failure data. Preliminary work in the field has been discussed in Appendix IV.

Conclusions and Recommendations

It has been demonstrated that an accurate description of fatigue is possible by means of stochastic model. A simple model has been developed This model can be quantitatively estimated. The model has been applied to develop a procedure for a reliability-based cost-effective fail-safe design for aircraft structures. In particular, reliability improvement due to inspection and maintenance has been considered.

Deterministic design procedures that do not consider the involved uncertainties usually result in an over design. This results in an increased weight that affects both cost and performance. Furthermore, risks involved in a deterministic design are not known. On the other hand, the reliability-based design that uses a stochastic model considers the uncertainties that are consistent with the model. Risks in a design can be assessed. consistent with the model considered. Such a procedure usually results in lower weight than deterministic designs. This results in low operating cost and better performance of the aircraft . A very costly item in owning and operating an aircraft is the inspection and maintenance during the life of the aircraft. As has been demonstrated in the project an optimum schemes can be developed by using a stochastic model for fatigue and considering the reliability improvement due to inspection and repair. Methods of including such reliability improvement at the design stage has also been discussed.

The following further investigations are suggested in the point of view of the practical application of the developed procedures.

1. Development of different types of stochastic models so that the user has a choice depending on the particular application. It is necessary that all uncertainties be properly included in the model. Different and more accurate methods of quantitative estimation and verification of the model are needed.
2. It is also necessary to develop simple optimization techniques to include the combination of discrete inspection costs with other costs. This is necessary to avoid the difficulty with local minimums.and provide a simple practical procedure.
3. The developed procedures should be modified to include multiple locations, a nd multiple cracks.
4. It appears as though cold working process will be used to improve the fatigue life of most existing and future metal aircraft. Probabilistic model for failure of such structures that includes both the life improvement and the stress corrosion susceptibility has not yet been developed. Such models are essential to fully take advantage of the cost and weight savings potential offered by the cold working processes.
5. In the point of view of increasing fuel costs, present levels of performance can be maintained only by using a material that has a higher strength to weight ratio than that offered by present aircraft structural materials. Advanced composites have such a potential. Mechanical behavior and failure modes of these advanced composites are different from that of metals. Instead of developing a deterministic design procedure and then modifying the procedures to develop probabilistic procedures,

reliability-based design procedure should be developed from the very beginning. By such a process the weight saving potential of advanced composites can be explored completely. This needs modification of the project to adopt to failure modes of composites.

6. Development of more accurate cumulative damage estimation techniques are essential for both metal and composite aircraft.

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Appendix I

In this appendix, a method of estimation of the static fatigue failure P_{fs} has been discussed. This fatigue failure is possible when the external loading exceeds the residual strength of the structure and the crack size a is less than a_c. By defining a quantity 's' in the following way

$$S = \sigma_R / \sigma_L \quad , \quad \sigma_L = L_{max} / t \qquad (1)$$

The probability of static fatigue failure can be defined as the probability of s being less than or equal to 1. Alternately, reliability against the static fatigue failure can be defined as R_s

$$R_S = P[S \geq 1] = P[\sigma_R / \sigma_L \geq 1] \qquad (2)$$

This probability can be evaluated from the following integral if the marginal probability density functions of σ_R and σ_L are given.

$$R_S = \int_1^\infty \int_{R_Z} |Z/S^2| \, f(Z) \, g(Z/S) \, dZ \, dS \qquad (3)$$

In this equation, f and g are the marginal probability density functions of σ_R and σ_L respectively, Z is an auxiliary variable and R_Z is the range space of Z. The integral given in Equation (3) is difficult to evaluate.

Instead of evaluating the integral of Equation (3), the following alternate procedure can be adopted to evaluate the static-fatigue reliability R_s[29-33]. The generalized Chebychev inequality is employed to determine the reliability R_s. For any shape of density function h(s), the probability that the random variable s lies within a range $(d - \delta) \leq s \leq (d + \delta)$ is given by the following inequality[34]:

$$P[(d-\delta) \leq S \leq (d+\delta)] \geq 1 - \frac{1}{\delta^2} E[S-d]^2 \qquad (4)$$

in this equation $E[\cdot]$ denotes the expectation operation, 2δ is the width of the strip and d is any particular value of s. The lower limit of s namely, $(d-\delta)$ is unity, i.e. $\delta = d - 1$. Substituting these limits in Equation (4),

$$P[1 \leq S \leq 2d-1] \geq 1 - \frac{1}{(d-1)^2}\left[E(S^2) - 2d\,E(S) + d^2\right] \qquad (5)$$

Now, recognizing that

$E(S) = \bar{S}$, the mean value of S, and

$E(S^2) = \sigma_S^2 + \bar{S}^2$

the equation(5) reduces to the following form after using Equation (2):

$$R_S \geq 1 - \frac{1}{(d-1)^2}\left[\sigma_S^2 + (\bar{S} - d)^2\right] \qquad (6)$$

For R_s to be a maximum it is necessary that[29, 32]

$$\frac{\partial R_S}{\partial d} = 0 \quad , \quad \frac{\partial^2 R_S}{\partial d^2} < 0 \qquad (7)$$

From the first of Equation (7),

$$\hat{d} = \bar{S} + \sigma_S^2/(\bar{S}-1) \qquad (8)$$

From the second of Equation (7) and (8)

$$\frac{\partial^2 R_S}{\partial d^2} = -2\,(\bar{S}-1)^4 / \left[(\bar{S}-1)^2 + \sigma_S^2\right] \qquad (9)$$

which is negative for all $\bar{S}$ and σ_S^2.

Substituting for d from Equation (8) in Equation (6) it follows that

$$R_S \geq \frac{(\bar{S}-1)^2}{\sigma_S^2 + (\bar{S}-1)^2} \qquad (10)$$

Appendix II

Numerical Example:

The problem is to design a stiffened panel subjected to a given random loading. The panel can have a central crack extending through the thickness. Also, the panel will be subjected to periodic maintenance inspections with attendant repairs of the crack when possible. Thus, the design variables involved can be categorized as follows:

(1) Material parameters

(2) Geometrical parameters and

(3) Maintenance parameters.

The design problem therefore consists of (1) selecting the optimum material from a given set of different materials, (2) selecting the optimum stringer spacing and thickness, and (3) selecting the optimum number of periodic inspections.

The following are assumed to be given and the designer has no choice in these variables

	SI units
ω = 20.0"	0.51m
h = 15.0	0.38 m
α = 7.5	7.5
β = 48,000 cycles	4.8×10^4 cycles
T_D = 6.0×10^5 cycles	6.0×10^5 cycles
R_b = 99.95%	99.95%

In the above set 'ω' and 'h' are the overall dimensions of the panel . The quantity α and β characterize the Weibull model for crack initiation of 0.005 inches. The design life T_D is to be 6.0×10^5 cycles. The

reliability restriant R_b should be 0.9995. The material properties are as follows:

For 7075T6 Aluminum Alloy

$K_{1c} = 68,000$ lb./in $^{3/2}$

$C = 5 \times 10^{-13}$

$n = 3$

For 2024-T3 Aluminum Alloy

$K_{1c} = 83,000$ lb/in$^{3/2}$

$C = 3 \times 10^{-13}$, $n = 3$

A computer program has been written to obtain the probability of failure for each selected thickness, stringer spacing, material and the number of periodic inspections N during the design life. This information is later used in another computer program to obtain the expected cost or weight function. The design variables that meet the minimum expected cost or weight function subject to reliability constraints are selected. The following tables illustrate representative results and the selected design variables and the material.

For the first material, i.e., 2026-T3 the overall minimum occurs for 6 periodic inspections, 3.3" stringer spacing and sheet thickness of 0.105". For 7075-T6, the overall minimum occurs for 6 periodic inspections, 3.5" stringer spacing and 0-103" thickness when both minimums were compared, 7075-T6 has the lower minimum weight at 6 inspections, 3.3" stringer spacing and 0.103" sheet thickness. Hence, 7075-T6 would be the selected material. All the details of the calculations will be published in the Ph.D. thesis of Mr. B. Uppaluri and in a journal.

Appendix III

Multiple integration with respect to one independent variable was needed in integrating the equations (7) and (8) of the main text to obtain $P(k\Delta\ell, t)$. Such a technique is also needed in many other engineering problems. Hunter[35] developed a method of numerical multiple integration and called it "the integrating matrix method". He applied the technique to forced vibration problem of helicopter rotor blades. In Hunter's method, the derivation of the integrating matrix consisted of dividing the range of integration into N intervals of equal size and N+1 points. At each of the N+1 points, the values of the integrand were obtained and represented in a column matrix $\{f_r\}$. The functional variation of the integrand in each interval was represented by an r^{th} degree polynomial. In order to obtain the values of the integral, an r^{th} degree integrating matrix $[I_r]$ was constructed b using Newton's interpolation formula. By multiplying the integration matrix and the integrand column matrix, the values of the integral were obtained. For multiple integration, the integrand matrix f_r was repeatedly multiplied by the integrating matrix $[I_r]$. For example,

$$\int_0^{x_1}\int_0^{x}\int_0^{x} f(x)\,dx\,d\tilde{x}\,d\hat{x} \simeq [I_n][I_n][I_n]\{f_n\} \qquad (1)$$

Improvement

The mathematical motivation for the improvement is the fact that when an 'r^{th}' degree polynomial is integrated an '$(r+1)^{th}$' degree polynomial is obtained. Thus, the improvement suggested is that the degree of the integrating matrix be increased by one after each integration is a multiple integral. For example,

$$\int_0^{x_1} \int_0^{x} \int_0^{x} f(x)\, dx\, d\bar{x}\, d\hat{x} \simeq [I_{n+2}][I_{n+1}][I_n]\{b_n\} \quad (2)$$

The improved method was applied for the following problems:

(i) Multiple Integration of an algebraic function $0 < x < 20$

(ii) Forced Vibration response of a Canilever beam

(iii) Free vibration of Canilever beams

The results were compared with the exact solutions.

In the first example, a constant function f(x) = 1.0 was successively integrated four times using a second degree integrating matrix and number of divisions N = 20. The percent error ranged from 200.0 at x = 1 to 0.5 at x=20. The improved method was employed with the same N = 20, but with integrating matrices of degree 2, 3, 4, and 5 successively. The percent error was zero all through the range of integration.

For the forced vibration problem the span was divided into five equal intervals (N = 6) and a second degree integrating matrix is employed four times consecutively. The percent error ranged from 6.4 at 1/5 span to 0.3 at 5/5 span. The improved technique with the same N = 6 but increasing degree of integrating matrix from two resulted in a maximum percent error of only 0.03.

For the free vibration problem less than 1 percent error in natural frequency and/or less mean square error in mode shape was obtained at a lower number of spanwise divisions than in the case when the integrating matrix was not altered. Also, the mean square error in the modeshape compared to the exact mode shape for any mode was less in the improved method than in the method of Hunter.

The difference between the two methods decreased as the degree of the starting integrating matrix is increased. All the results will be published. At present the manuscript is being prepared.

Appendix IV

Introduction

Analysis of variance is a means of determining the homogeneity of a large collection of data that have been formed by lumping together several small groups of data. The small groups are denoted as "sub groups" and the variation between them as "variation between subgroups". The name, analysis of variance, itself stems from an analysis in which the total variation in the entire data is partitioned into component parts. These components are used to develop a test statistic.

The total variation is expressed by the total corrected sum of squares, i.e.

$$SS_T = \sum_{i=1}^{a} \sum_{j=1}^{m_i} X_{ij}^2 - T_{..}^2/N \qquad (1)$$

In this equation

a is the number of treatments

X_{ij} is the data point

$T_{..}$ is the total sum of data points

N is the total number of data points, and

n_i is the number of data points in 'i'th treatment.

The total variation SS_T can be split up into two components as follows:

$$SS_T = SS_A + SS_E, \qquad (2)$$

The term SS_A is variation between subgroups and SS_E is variation within subgroups. Then, the following table is constructed to facilitate the analysis of variance.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Sum of Squares	F
Between sub-groups	SS_A	a-1	SS_A (a-1)	$\frac{SS_A/(a-1)}{SS_E/(N-a)}$
Within sub-groups	SS_E	N-a	$SS_E/(N-a)$	
Total	SS_T	N-1		

The value in the last column is compared with the critical F value at a given percent of significance and degrees of freedom of (a-1) and (N-a) respectively. The data is homogeneous if the F value is less than the critical F value [36].

If the above analysis of variance indicates that the data is non-homogeneous, then it is desirable to find out which of the subgroups form a homogeneous set of data. For this purpose, Duncan's multiple range test [37] can be employed. It consists of comparing the modified difference between the various means $(m_i - m_j)'$ with the corresponding critical value R'_p. The modified means are calculated from the following expressions

$$(m_i - m_j)' = (m_i - m_j)\, a_{ij} \qquad (3)$$

$$a_{ij} = \left(\frac{2r_i r_j}{r_i + r_j}\right)^{1/2} \qquad (4)$$

where r_i, r_j are the number of replications in each group. The critical values can be calculated from Table II of Duncan's [37] paper. Then all

the possible groups are subjected to Duncan's test and those groups whose modified mean does not exceed the critical value of R'_p belong to one homogeneous set of data.

Application

The procedure that has been discussed in the preceding paragraph is used to analyze the fatigue failure data from a specific fleet of aircraft. The objective is to investigate if the fatigue failure data from several critical regions can be lumped together. If it is possible to lump the data together a small number of probability distributions can be used to describe the fatigue failure of the entire structure. It is also possible to use the system of lumping to do large number of inspections at a few representative locations.

The particular aircraft under consideration has 92 fatigue critical regions. Investigations show that the station group (2 to 15), (33 to 38), (41 to 46) and (89 to 92) can be lumped together. Analysis of variance tests indicate that these subgroups form a homogeneous set of fatigue data. The station groups (1-92), (61-70) and (71-80) cannot be lumped together because the test results show that their data varies significantly. These results are quantitatively presented in the following table.

Group	Variance		Result
	Within Groups	Between Groups	
(2-15)	0.9490	0.7785	No Significant Variation
(33-38)	0.2680	0.5972	No Significant Variation
(41-46)	0.5229	0.4460	No Significant Variation
(41-46, 89-92)	0.8026	0.3890	No Significant Variation
(89-92)	0.8457	0.4224	No Significant Variation
(61-70)	0.6846	3.7367	Data Varies Significantly
(71-80)	0.6753	1.8720	Data Varies Significantly
(33-38, 41-46, 61-72, 89-92)	0.7488	3.8284	Data Varies Significantly
1-92	0.7651	1.4761	Data Varies Significantly

Complete details will be published in a Journal.

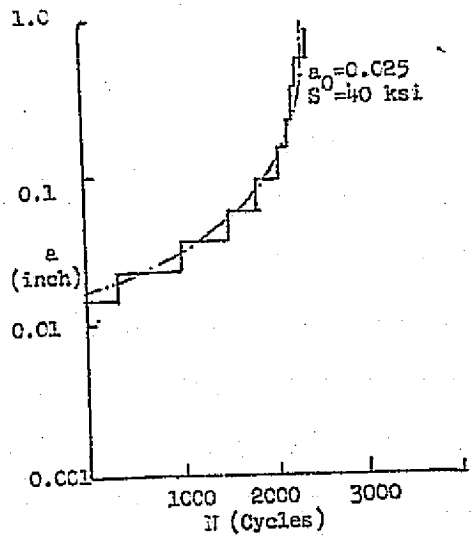

Figure 1 Crack Length versus Growth Cycles

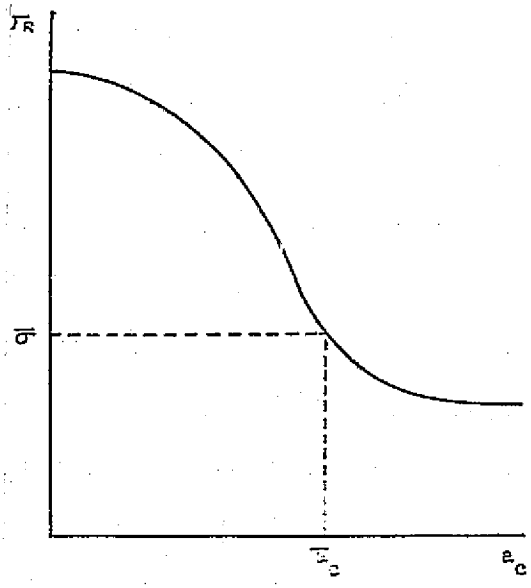

Figure 4 Residual Strength Variation

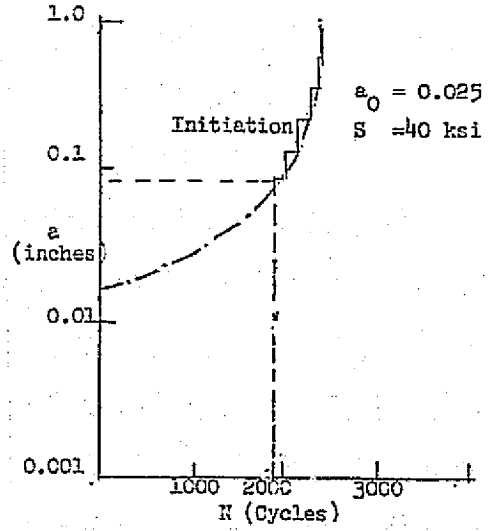

Figure 2 Crack Length versus Growth

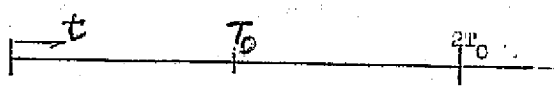

Figure 5 Inspection Schedule

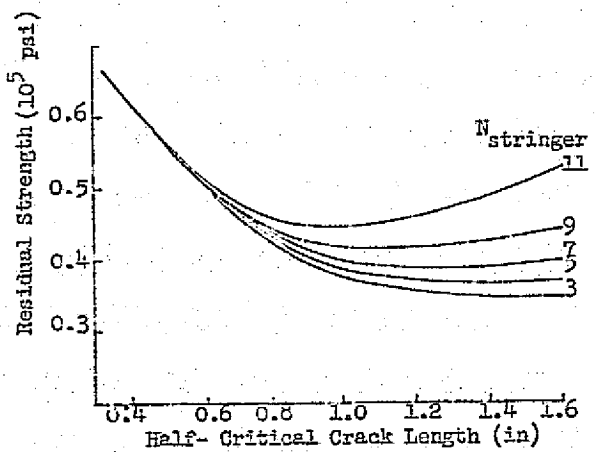

Figure 6
Residual Strength Variation

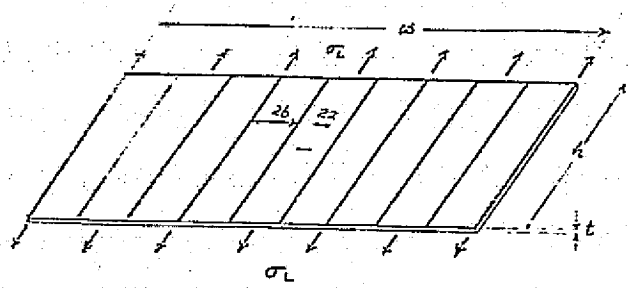

Figure 3 Panel Configuration

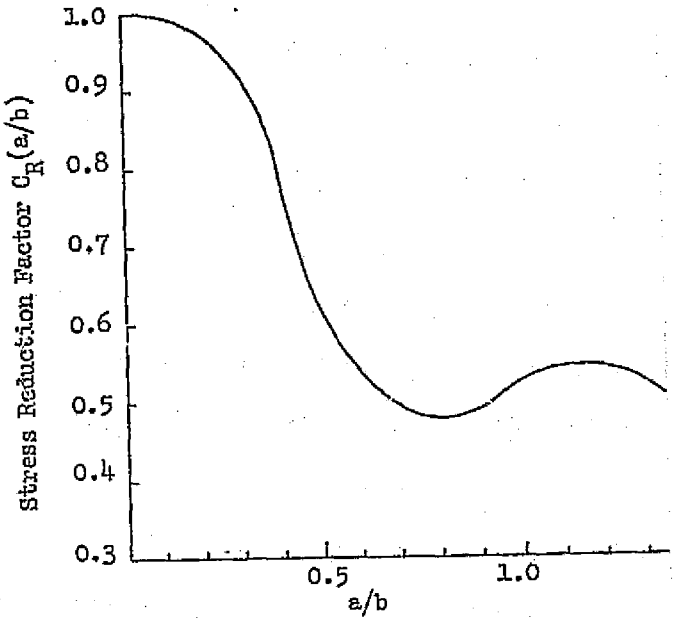

Figure 7 Stress Reduction Factor

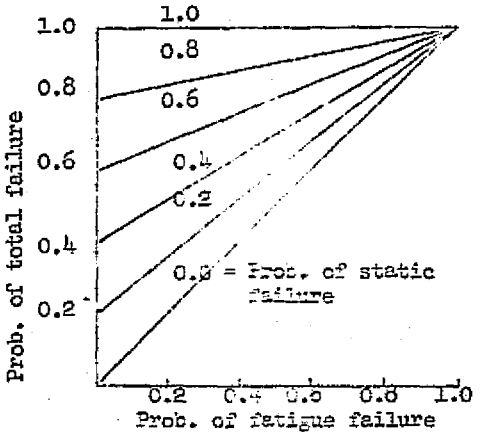

Figure 9 Static failure-Fatigue failure Relationship

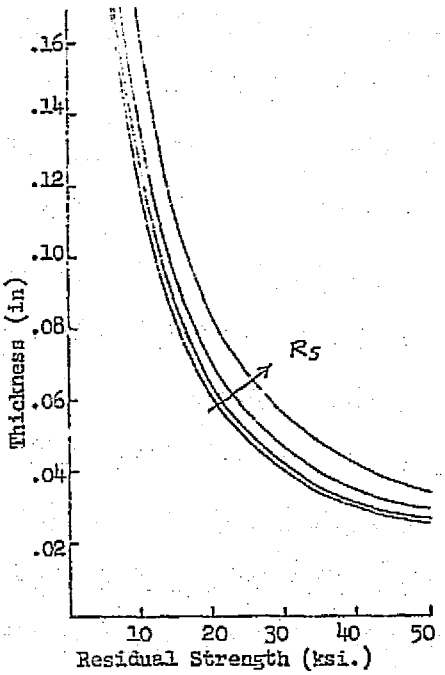

Figure 8 Static Reliability Variation

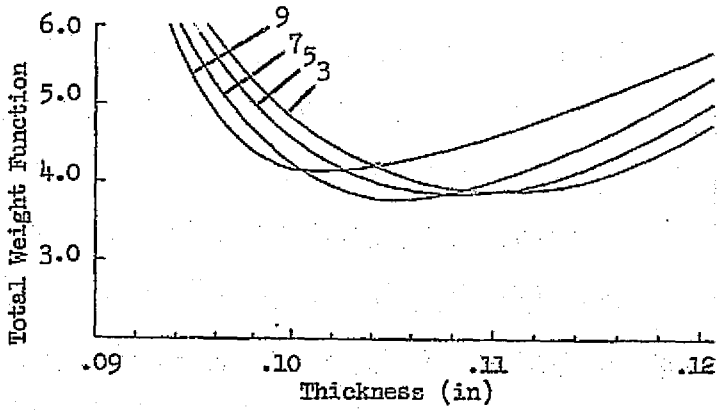

Figure 10 Minimization Curves

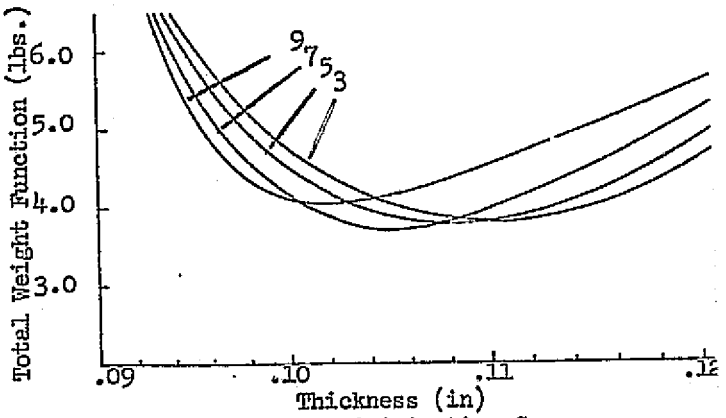

Figure 11 Minimization Curves

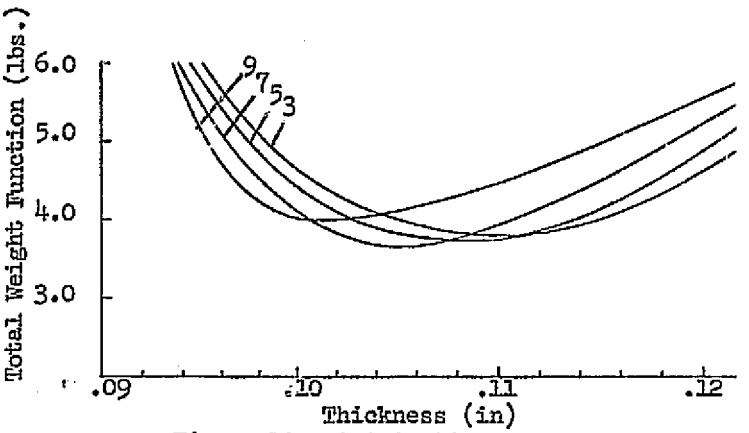

Figure 13 Minimization Curves

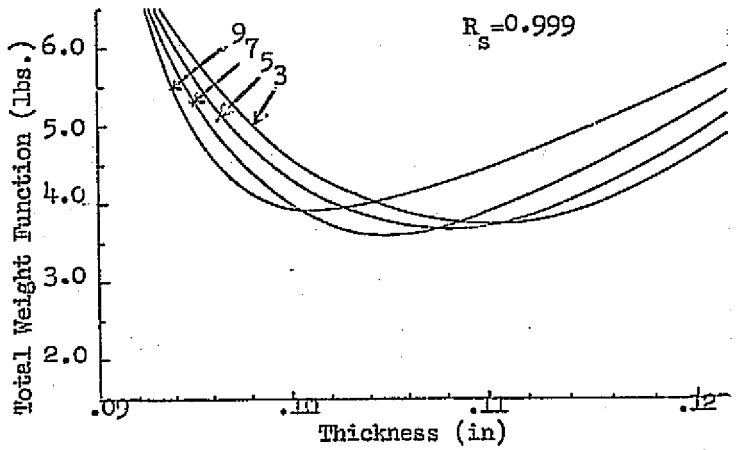

Figure 12 Minimization Curves

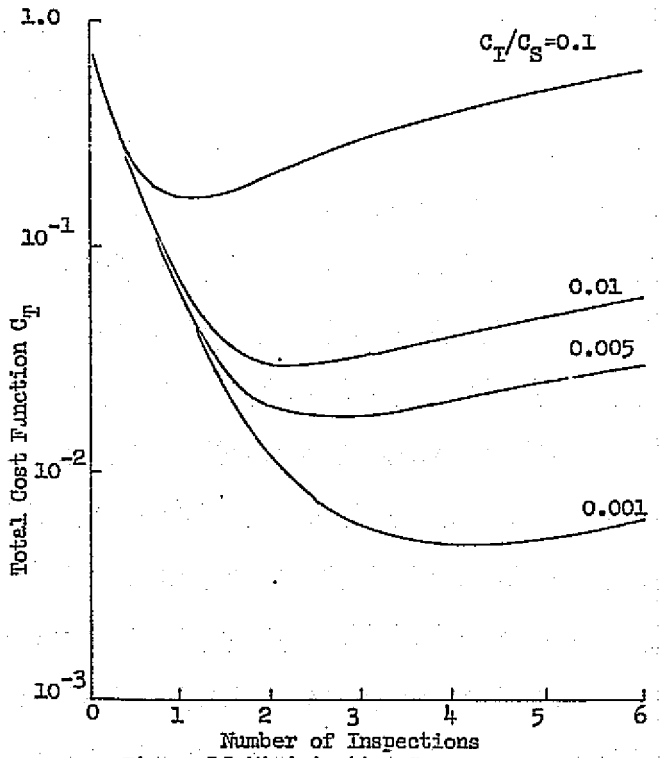

Figure 15 Minimization Curves

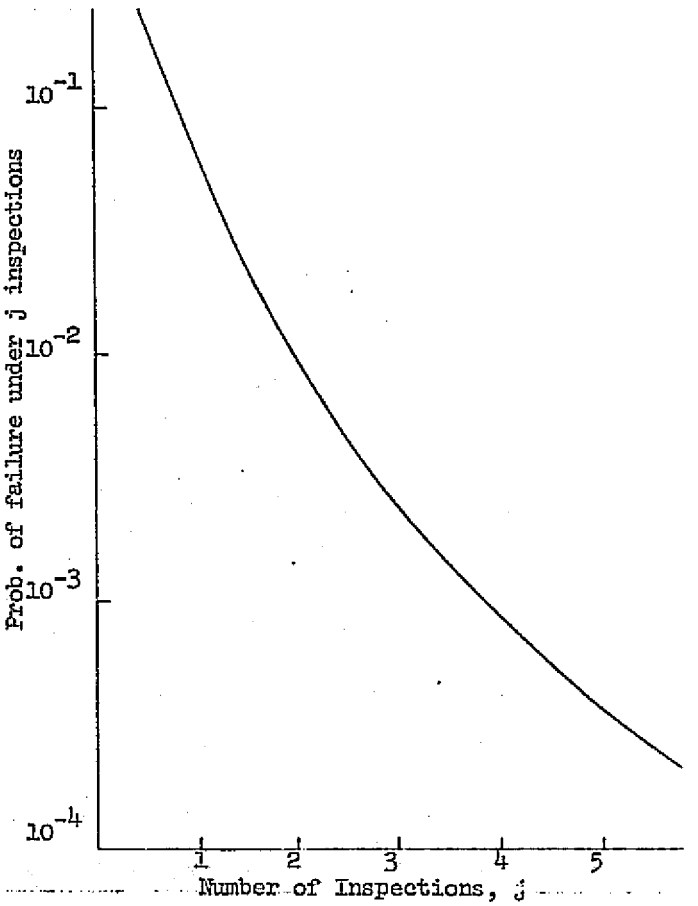

Figure 14 Reliability Variation with Number of Inspections